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DRAWING OF A FREE JET BY A ROTATING ROLL

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Viscous fluid flow in the initial section of the contact of a free jet with a rotating roll is considered.

In the technology of polymer processing and rheology of longitudinal flows, a roll rotating at a constant velocity is used to draw a free jet. In the description of a free jet flow, it is customary to assume that the flow terminates when the jet touches the roll surface and the fluid particles acquire a velocity equal to the circumferential velocity of the roll [1].

On studying the drawing of a jet of polyoxiothylene solution (a non-Newtonian fluid with well-defined elastic properties) A. N. Prokunin revealed that the jet velocity near the contact point with the roll is smaller than the circumferential velocity of the roll by a factor of 1.5–3, i.e., the jet effectively slips relative to the roll [2]. Namely the ability of an extended free jet of a viscoelastic fluid to slip at a side contact with a wall is responsible for the effect of normal stresses described in [3].

Slipping of flat jets of polymer melts along the surface of pulling-in rolls is analyzed in [4]. A great influence of friction on the stability of the formation process and on the qualitative characteristics of the obtained flat films is noted.

The available approaches (for example, the problem of plate withdrawal from a fluid [5] or the Euler exponential law of the change in the tension of a belt along the roll circumference [6]) are not applicable to the considered flow.

The described facts determine the advisability of a more detailed consideration of hydrodynamic interaction of the side surface of a free extended jet with a solid impermeable surface. In the present paper an attempt is undertaken to analyze, in the Stokes approximation, hydrodynamic interaction in the "extended jet–take-off roll" system assuming that the flow continues in the fluid film found on the solid surface.

The flow scheme is given in the figure. The fluid jet with an initial velocity v_s, escaping from the nozzle is subjected to uniaxial extension by the take-off roll. The extension zone has length l. The axial velocity of the fluid in the section of the roll contact is v_0, and the tensile stresses are $\sigma_{xx} = T$. The roll of radius R rotates with velocity ω.

We consider a fluid film on the roll. Directly at the contact instant the thickness of the flat jet is δ_0, and at a large distance from the contact point it is δ_1. Assuming $\delta_1 << R$, we neglect the surface curvature of the roll and consider the flow in a Cartesian system of coordinates whose origin is located at the contact point (a rest coordinate system according to Euler). The y axis is directed along the radius and the x axis coincides with the rotation direction and lies on the roll surface. The fluid is Newtonian and highly viscous. Surface tension, inertial forces, forces of aerodynamic resistance, and eigenweight are neglected. The sticking condition $v_x = \omega R$, $v_y = 0$ is taken on the surface. The film thickness is uniform over the width. Fluid particles move along trajectories that lie in planes normal to the roll axis, and $v_z = 0$, $\partial/\partial z = 0$. The flow is steady-state and isothermal. On the free surface $\delta = \delta(x)$, continuity of the normal and the absence of tangential stresses are assumed. The pressure on the free surface is $P_0 = 0$. Prior to contact with the roll, the flow in the jet is quasi-one-dimensional, i.e., is characterized by axial velocity and tensile stresses that are uniform across the section [7]. In the initial section of contact with the roll, the flow is two-dimensional and besides nonuniform normal stresses there appear tangential ones caused by the hydrodynamic effect of the roll wall. The assumed length of the section of hydrodynamic interaction is commensurate with the film thickness.

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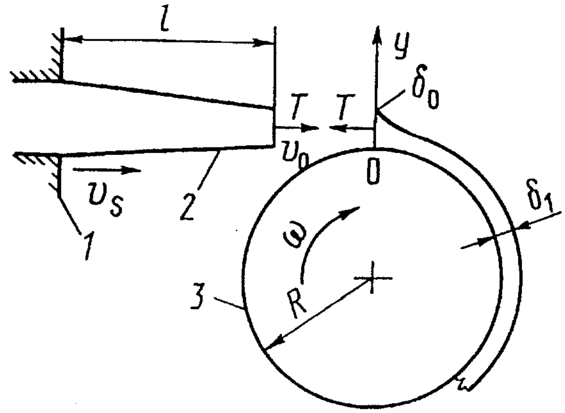

Fig. 1. Flow diagram: 1) nozzle; 2) free jet; 3) roll.

With allowance for the adopted assumptions, fluid flow on the roll surface within the range $0 \le x \le +\infty$, $0 \le y \le \delta(x)$ is described by the equations

$$\frac{1}{\mu}\frac{\partial P}{\partial x} = \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}, \quad \frac{1}{\mu}\frac{\partial P}{\partial y} = \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2}, \tag{1}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0, \tag{2}$$

$$x = 0, \quad T = -P + 2\mu\frac{\partial v_x}{\partial x}, \quad v_x = v_0, \quad \delta = \delta_0, \tag{3}$$

$$x = \infty, \quad v_x = \omega R, \quad P = 0, \quad v_y = 0, \quad \delta = \delta_1, \tag{4}$$

$$y = 0, \quad v_x = \omega R, \quad v_y = 0, \tag{5}$$

$$y = \delta, \quad \tau_{xy} = \mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) = 0, \quad \sigma_{yy} = -P + 2\mu\frac{\partial v_y}{\partial y} = 0. \tag{6}$$

Integrating Eq. (2) over the film thickness, we obtain an equation for the profile of the free surface

$$\frac{\partial}{\partial x}\int_0^\delta v_x dy - v_x(\delta)\frac{d\delta}{dx} + v_y(\delta) = 0. \tag{7}$$

We determine the velocity components in terms of the stream function

$$v_x = \frac{\partial \Psi}{\partial y}, \quad v_y = -\frac{\partial \Psi}{\partial x}. \tag{8}$$

Here Eqs. (1) take the form

$$\frac{1}{\mu}\frac{\partial P}{\partial x} = \frac{\partial^3 \Psi}{\partial y \partial x^2} + \frac{\partial^3 \Psi}{\partial y^3}, \quad -\frac{1}{\mu}\frac{\partial P}{\partial y} = \frac{\partial^3 \Psi}{\partial x^3} + \frac{\partial^3 \Psi}{\partial x \partial y^2}. \tag{9}$$

Then, using Eq. (9) it is easy to obtain equations for P and Ψ:

$$\nabla^2 P = 0, \quad \nabla^4 \Psi = 0. \tag{10}$$

We find the velocity and pressure fields for a semi-infinite band $0 \le y \le \delta_1$, $0 \le x \le +\infty$ under the assumption of small deviations of the surface $\delta_0 - \delta_1 << \delta_1$. We transfer the boundary conditions from the δ line to the δ_1 line.

Solutions in the form of [8]

$$\Psi = (C_1 \sin \alpha y + C_2 \cos \alpha y + C_3 \alpha y \sin \alpha y + C_4 \alpha y \cos \alpha y)\, e^{\alpha x} + C_5 y,$$

$$P = \mu A_1 (\sin \alpha y + A_2 \cos \alpha y)\, e^{\alpha x}.$$

satisfy Eqs. (10). The constants $C_1 - C_5$ are found from the boundary conditions. Using condition (5), we obtain $C_2 = 0$, $C_1 = -C_4$, and $C_5 = \omega R$. From the condition for tangential stresses (6) we have $C_1 = C_3$ (tan $\alpha\delta_1 - 1/\alpha\delta_1$). For the stream function we can write

$$\Psi = C_3 \left[\left(\tan \alpha\delta_1 - \frac{1}{\alpha\delta_1} \right) (\sin \alpha y - \alpha y \cos \alpha y) + \alpha y \sin \alpha y \right] e^{\alpha x} + \omega R y. \quad (11)$$

The last term allows for translational fluid flow along the roll surface.

The constants A_1 and A_2 are found by any of Eqs. (9). We have $A_1 = -2C_3\alpha^2$, $A_2 = (1/\alpha\delta_1 - \tan \alpha\delta_1)$. The pressure is described by the relation

$$P = -2\mu\alpha^2 C_3 \left[\sin \alpha y + \left(\frac{1}{\alpha\delta_1} - \tan \alpha\delta_1 \right) \cos \alpha y \right] e^{\alpha x}. \quad (12)$$

The eigenvalues of the problem are determined from the condition of normal stresses on the surface (6). Having assumed $\alpha = -\lambda/\delta_1$ and substituted (11) and (12) into σ_{yy}, we obtain the equation $\pm\lambda = \cos \lambda$, whence $\lambda_1 = 0.739$. We restrict ourselves to an aperiodic approximation for the flow. The existence of only one eigenvalue is due to the boundary conditions with respect to y.

The solution in the form of (11) does not satisfy the initial condition (3); therefore, we shall call for its integral-mean fulfillment

$$x = 0, \quad \frac{1}{\delta_0} \int_0^{\delta_0} v_x dy = v_0,$$

whence

$$C_3 = \frac{\delta_0 \lambda (v_0 - \omega R)}{1 - \lambda \tan \lambda}. \quad (13)$$

Moreover, the integral form of the boundary conditions (3) for v_x and T smoothes the singularity of the contact point, where $v_x(x = -0, y = 0) = v_0$, $v_x(x = +0, y = 0) = \omega R$, and the velocity gradient is infinite. It is obvious that under real conditions the hydrodynamic effect exerted by the roll on the jet starts before it contacts the solid surface.

The equation for a free surface (7) will be expressed in terms of the stream function:

$$\frac{\partial}{\partial x} (\Psi |_0^\delta) - \frac{d\delta}{dx} \frac{\partial \Psi}{\partial y}\bigg|_\delta - \frac{\partial \Psi}{\partial x}\bigg|_\delta = 0. \quad (14)$$

In (3) and (14) we use the condition $\delta_0 - \delta_1 << \delta_1$, and in trigonometric functions we assume $\delta_0/\delta_1 \approx \delta/\delta_1 \approx 1$.

From Eq. (14) we obtain a first-order differential equation for the thickness of the fluid film on the roll

$$\frac{d\delta}{dx} = \frac{B\lambda}{B - \lambda} \left(\frac{\delta}{\delta_1} - 1 \right),$$

where $B = (1/\lambda - \tan \lambda)(\lambda^2 - \sin \lambda) + \lambda \sin \lambda$. With allowance for the initial condition (3), its solution is

$$\bar{\delta} = \frac{\delta - \delta_1}{\delta_0 - \delta_1} = \exp\left(-1.097\frac{x}{\delta_1}\right). \tag{15}$$

According to (15), the length of the flow zone on the roll is determined only by the film thickness at infinity δ_1. At $x = 3\delta_1$, $\bar{\delta} = 0.037$ and the flow virtually terminates.

The initial stress T will be found from the condition of normal stresses (3) in integral form

$$T = \frac{1}{\delta_0}\int_0^{\delta_0}\left(2\mu\frac{\partial^2\Psi}{\partial x\partial y} - P\right)dy.$$

Upon integrating with allowance for (11)-(13), we obtain

$$T = \frac{2\lambda^3(\sin\lambda - \lambda\tan\lambda)}{1 - \lambda\tan\lambda}\,\frac{\mu(\omega R - v_0)}{\delta_0},$$

or

$$T = 3.347\frac{\mu(\omega R - v_0)}{\delta_0}.$$

We introduce multiplicity of drawing on the roll $K_2 = \omega R/v_0$. It follows from the condition of flow continuity in the initial section and at infinity that $\delta_0 v_0 = \delta_1\omega R$, or $\delta_0 = K_2\delta_1$. Here we assume that the jet width is constant. It can be written for the initial stress that

$$T = 3.347\frac{\mu\,\omega\,R(K_2 - 1)}{\delta_1 K_2}. \tag{16}$$

The stress T is caused by jet extension between the nozzle and the take-off device. It is interesting to estimate the relation between the conditions of free jet drawing and fluid film deformation on the roll surface. For a free isothermal jet we take an exponential distribution of axial velocity [9]

$$v = v_s\exp\left(\frac{x}{l}\ln K_1\right),$$

where x is the distance from the nozzle. Tensile stresses in the take-off cross-section are

$$T = 3\mu\left.\frac{\partial v}{\partial x}\right|_{x=l} = \frac{3\mu v_s K_1\ln K_1}{l}. \tag{17}$$

There is a relation between the multiplicities of drawing K_1 and K_2

$$K = K_1K_2, \tag{18}$$

where $K = \omega R/v_s$ is the total multiplicity, $K_1 = v_0/v_s$.

From the combined consideration of (16)-(18) we obtain for the multiplicity of fluid film pulling on the roll surface

$$K_2 = 1 + \frac{3}{3.347}\,\frac{\delta_1}{l}\ln K_1.$$

For Newtonian fluids the effect of drawing on the roll is greatest in the case of a thick film δ_1 and at small lengths of the free jet l.

NOTATION

P, pressure; μ, viscosity; x, y, z, coordinates; v_x, v_y, v_z, velocity components; T, tensile stress in jet at the moment of contact with roll; δ, current fluid film thickness on roll; δ_0, initial film thickness; δ_1, film thickness at large distance from contact point; $\bar{\delta}$, dimensionless film thickness; τ_{xy}, tangential stress; σ_{xx}, σ_{yy}, normal stresses; Ψ, stream function; α, λ, constants; R, roll radius; v_s, axial velocity of fluid near nozzle; v_0, axial velocity at moment of jet contact with roll surface; l, length of section of free jet extension; K, K_2, K_1, total multiplicity, multiplicity of drawing on roll and in the zone of elongation flow.

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